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A semi-analytical solution for forced vibrations response of rectangular orthotropic plates with various boundary conditions †

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Abstract

Orthotropic plates form an essential part of many marine, aerospace, and automobile structures. The recent increase in the use of composite materials for plate-type structures intensified the need for solutions utilizing orthotropic plates. These structural components, in many instances, are subjected to vibration. The main purpose of this paper is to present a semi-analytical analysis of the response of orthotropic plates to forced vibrations with different combinations of boundary conditions subjected to a general distributed excitation. The solution of the partial differential equation was reduced to an iterative sequential solution of ordinary differential equations using extended Kantorovich method. The efficiency and accuracy of the proposed method were examined through comparison with available literature and was in good agreement with them.

Keywords: Extended Kantorovich method; Forced vibrations; Orthotropic plates

1. Introduction

Rectangular plates have wide applications in civil, mechanical, aeronautical, and marine structures. Engineers often use orthotropic rectangular plates when finding solving their work predicaments. In architectural engineering, for instance, reinforced-concrete slabs with one- or two-way joists are used for floor systems in buildings. In civil engineering, highway bridge decks usually consist of plates stiffened with rectangular, triangular, or trapezoidal ribs. The use of stiffened plates is especially indispensable in ship and aerospace structures; the hull, deck, bottom and superstructure of a ship could be considered as orthotropic plates. Similarly, in flight structures, the wings and fuselage consist of skin with an array of stiffening ribs.

Among its various aspects, vibrations have drawn much attention in terms of the structural performance of orthotropic plates. In many instances an orthotropic plate should support a motor, which involve the consideration of dynamic disturbances produced by time-dependent external forces. The objective of this study is to determine the effect of vibration, and investigate the vibration characteristics of orthotropic plates, in order to develop accurate and reliable designs.

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Closed-form solutions of dynamic responses of plates were presented mainly for a few edge boundary conditions, such as those simply supported at four edges (Navier-type solutions) or at parallel edges (Levy-type solutions) [1]. These methods can be extended to orthotropic plates [2]. For other types of boundary conditions, many efforts were devoted to develop approximate methods, such as the Rayleigh-Ritz method [3, 4], superposition method [5, 6], Green's function method [7], Lagrangian multiplier technique [8], and the finite element method [9].

In spite of its importance in terms of applicability to real structures, the number of studies that focused on the forced vibration analysis of orthotropic plates is relatively limited. Similarly, in all these studies, as well as in many others, the influences of general loading and boundary conditions were not accounted for. Numerical methods, such as finite element method (FEM), needs much effort in modeling the geometry and in applying the general load, which sometimes are not needed especially in the early stage of the design process. In these cases, an approximate, quick, and easy-to-apply solution is beneficial.

A very efficient approximate closed-form solution is the so-called Extended Kantorovich Method (EKM) which was introduced by Kerr [10]. The Kantorovich method [11], which reduces partial differential equation to ordinary differential equations (ODE), was extended by Jones and Milne [12], and Bhat et al. [13] to study the free vibration of isotropic rectangular plates. Dalaei and Kerr [14], and Bercin [15] used the method as reference [12] to obtain natural frequencies of fully clamped orthotropic thin plates. Sakata et al. [16] applied the method as reference [13] to the vibration analysis of rectangular orthotropic plates.

The main purpose of the present work is to demonstrate an efficient and accurate application of the EKM for the forced vibration analysis of orthotropic rectangular plates with arbitrary boundary conditions under a general type of non- uniform distributed dynamic excitation. In this paper, dynamic displacement and bending moment amplitudes were determined. A thorough comparison with the available published results is presented for different cases with good agreement was observed.

2. Basic equations and boundary conditions

2.1 Governing equations

The plate and coordinate system are shown in Fig. 1.

An orthotropic material is characterized by the fact that the mechanical elastic properties have two perpendicular planes of symmetry, and the four elastic constants E_1, E_2, G, υ_1 are independent. E_1 and E_2 are Young's modulus, υ_1 and υ_2 are the corresponding Poisson ratios in these directions, and G is the shear modulus. The coefficient υ_2 can be determined from:

$$\frac{\upsilon_1}{E_1} = \frac{\upsilon_2}{E_2} \tag{1}$$

Orthotropic bending stiffness parameters are defined as:

$$D_1 = \frac{E_1 h^3}{12(1-\upsilon_1\upsilon_2)}, \quad D_2 = \frac{E_2 h^3}{12(1-\upsilon_1\upsilon_2)}$$

$$D_t = \frac{Gh^3}{12}, \quad D_3 = \upsilon_1 D_2 + 2D_t = \upsilon_2 D_1 + 2D_t \tag{2}$$

where h is the plate thickness. The governing differential equation for the forced vibration of orthotropic thin plate [2] is:

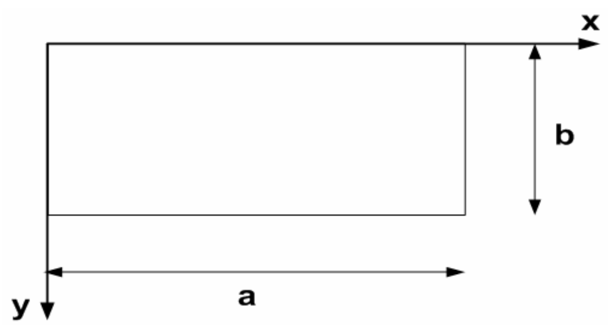

Fig. 1. Geometry of the plate.

$$D_1\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2D_3\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 w(x,y,t)}{\partial y^4}$$
$$+\rho h\frac{\partial^2 w(x,y,t)}{\partial t^2} = p_z(x,y,t) \tag{3}$$

where ρ is the mass density of the material per unit volume and p_z is the excitation force, which is expressed as:

$$p_z(x,y,t) = q(x,y)e^{i\omega t} \tag{4}$$

The deflection of plate is written as:

$$w(x,y,t) = W(x,y)e^{i\omega t} \tag{5}$$

The boundary conditions of the plate are

$x = cons$

Simply supported (S) $w = \dfrac{\partial^2 w}{\partial x^2} = 0$

Clamped (C) $w = \dfrac{\partial w}{\partial x} = 0$

$y = cons$

Simply supported (S) $w = \dfrac{\partial^2 w}{\partial y^2} = 0$

Clamped (C) $w = \dfrac{\partial w}{\partial y} = 0$ (6)

2.2 Application of the EKM

According to the extended Kantorovich method, deflection surface of the vibrating plate can be separated into two uncoupled function as:

$$w(x,y,t) = W(x,y)e^{i\omega t} = \xi(x)\psi(y)e^{i\omega t} \tag{7}$$

Here, $\xi(x)$ and $\psi(y)$ are the unknown functions of x and y is yet to be determined. The boundary conditions in the form of Eq. (7) can be written as

$x = cons$

Simply supported (S) $\xi = \dfrac{d^2\xi}{dx^2} = 0$

Clamped (C) $\xi = \dfrac{d\xi}{dx} = 0$

$y = cons$

Simply supported (S) $\psi = \dfrac{d^2\psi}{dy^2} = 0$

Clamped (C) $\psi = \dfrac{d\psi}{dy} = 0$ (8)

Substituting Eq. (7) into Eq. (3) yields the new form of governing equation in terms of separable functions:

$$D_1\psi \frac{d^4\xi}{dx^4} + 2D_3\frac{d^2\xi}{dx^2}\frac{d^2\psi}{dy^2} + D_2\xi\frac{d^4\psi}{dy^4} - \xi\psi\rho h\omega^2 = q(x,y) \tag{9}$$

Based on the general procedure of the Galerkin technique, multiplying Eq. (9) by the appropriate weighting function, ψ, and then assuming a prescribed function as an initial guess, and integrating over the entire length of the plate in the direction y leads to the first set of ODE as

$$F_1\frac{d^4\xi}{dx^4} - 2F_2\frac{d^2\xi}{dx^2} + F_3\xi = B \tag{10}$$

When the weighting function is ξ, in the same way, and with integration over the entire length of the plate in the x direction, the second set of ODE are obtained from

$$G_1\frac{d^4\psi}{dy^4} - 2G_2\frac{d^2\psi}{dy^2} + G_3\psi = I \tag{11}$$

The values of B, I, F's and G's are specified as:

$$F_1 = \int_0^b D_1\psi^2 dy\ ,\ \ G_1 = \int_0^l D_2\xi^2 dx$$
$$F_2 = \int_0^b D_3(\frac{d\psi}{dy})^2 dy\ ,\ \ G_2 = \int_0^l D_3(\frac{d\xi}{dx})^2 dx$$
$$F_3 = \int_0^b [D_2(\frac{d^2\psi}{dy^2})^2 - \rho h\omega^2\psi^2]dy$$
$$G_3 = \int_0^l [D_1(\frac{d^2\xi}{dx^2})^2 - \rho h\omega^2\xi^2]dx$$
$$B = \int_0^b q(x,y)\psi dy\ ,\ \ I = \int_0^l q(x,y)\xi dx \tag{12}$$

In summary, any arbitrary function for $\psi(y)$ was assumed as initial guess. Solving Eq. (10) gives first approximation of $\xi(x)$, and all constants in Eq. (11). Equation (11) leads to first approximation of $\psi(y)$. It can be seen that the first iteration is completed. Using the same iteration procedure, final form of functions $\psi(y)$ and $\xi(x)$ could be obtained.

3. Closed form solutions

In all iterations, for both ODEs (10) and (11) exact closed form solutions [17] can be expressed as:

$$\xi = \sum_{i=1}^4 A_i \exp(\lambda_i x) + \xi_p$$
$$\psi = \sum_{i=1}^4 B_i \exp(\mu_i y) + \psi_p \tag{13}$$

The first term in Eq. (13) is the general solution, and the second term is the particular solution. λ_i and μ_i ($i=1, ..., 4$) are roots of the following characteristic equations,

$$F_1\lambda^4 - 2F_2\lambda^2 + F_3 = 0$$
$$G_1\mu^4 - 2G_2\mu^2 + G_3 = 0 \tag{14}$$

3.1 Boundary conditions

The constants A_i and B_i (Eq. (13)) could be found based on the boundary conditions. For some cases, the computations are as follows:

All edges clamped

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ e^{\lambda_1 l} & e^{\lambda_2 l} & e^{\lambda_3 l} & e^{\lambda_4 l} \\ \lambda_1 e^{\lambda_1 l} & \lambda_2 e^{\lambda_2 l} & \lambda_3 e^{\lambda_3 l} & \lambda_4 e^{\lambda_4 l} \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} -\xi_p(0) \\ -\frac{d\xi_p}{dx}(0) \\ -\xi_p(l) \\ -\frac{d\xi_p}{dx}(l) \end{pmatrix}$$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ e^{\mu_1 b} & e^{\mu_2 b} & e^{\mu_3 b} & e^{\mu_4 b} \\ \mu_1 e^{\mu_1 b} & \mu_2 e^{\mu_2 b} & \mu_3 e^{\mu_3 b} & \mu_4 e^{\mu_4 b} \end{bmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} = \begin{pmatrix} -\psi_p(0) \\ -\frac{d\psi_p}{dy}(0) \\ -\psi_p(l) \\ -\frac{d\psi_p}{dy}(l) \end{pmatrix} \tag{15}$$

Combination of clamped and simply supported edges

To increase the applicability of this method, the SCCC, SSCC, SCSC, and CSSS boundary conditions are also considered. For example, the values of A_i and B_i for SSCC are as:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 \\ e^{\lambda_1 l} & e^{\lambda_2 l} & e^{\lambda_3 l} & e^{\lambda_4 l} \\ \lambda_1 e^{\lambda_1 l} & \lambda_2 e^{\lambda_2 l} & \lambda_3 e^{\lambda_3 l} & \lambda_4 e^{\lambda_4 l} \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} -\xi_p(0) \\ -\frac{d^2\xi_p}{dx^2}(0) \\ -\xi_p(l) \\ -\frac{d\xi_p}{dx}(l) \end{pmatrix}$$
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \mu_1^2 & \mu_2^2 & \mu_3^2 & \mu_4^2 \\ e^{\mu_1 b} & e^{\mu_2 b} & e^{\mu_3 b} & e^{\mu_4 b} \\ \mu_1 e^{\mu_1 b} & \mu_2 e^{\mu_2 b} & \mu_3 e^{\mu_3 b} & \mu_4 e^{\mu_4 b} \end{bmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix} = \begin{pmatrix} -\psi_p(0) \\ -\frac{d^2\psi_p}{dy^2}(0) \\ -\psi_p(l) \\ -\frac{d\psi_p}{dy}(l) \end{pmatrix} \tag{16}$$

3.2. Loading

Particular solutions of ODEs depend on the type of loading. For a general lateral dynamic load in the form of

$$p_z(x,y,t) = q(x,y)e^{i\omega t} = q_0 \mathrm{XT} e^{k_1 x + k_2 y} e^{i\omega t} \tag{17}$$

in which

$$\mathrm{X} = mx^4 + nx^3 + ox^2 + px + l$$
$$\mathrm{T} = Sin(ry) \quad \text{or} \quad \mathrm{T} = Cos(ry)$$

and m, n, o, p, l, r, k_1, and k_2 are arbitrary constants, particular solutions can be determined using the relation

$$\xi_p = e^{k_1 x} B_1 \sum_{i=1}^{5} U_i \Gamma_i \,,$$
$$\Gamma_i = \frac{d^{i-1}}{dx^{i-1}}(\mathrm{X}) \quad (i = 1,2,\ldots,5)$$
$$\psi_p = \frac{e^{k_2 y} I_1}{J_1^2 + (rJ_2)^2} \sum_{i=1}^{2} (-1)^{i-1} J_i \Phi_i \,,$$
$$\Phi_i = \frac{d^{i-1}}{d\theta^{i-1}}(\mathrm{T}) \quad (i = 1,2) \tag{18}$$

where B_1, I_1, $U's$ and $J's$ are:

$$B_1 = \int_0^{\alpha} q_0 \mathrm{T} e^{k_2 y} dy \,, \quad I_1 = \int_0^{l} q_0 \mathrm{X} e^{k_1 x} dx$$
$$U_1 = \frac{1}{S_1} \,, \quad U_2 = \frac{S_2}{S_1^2}$$
$$U_3 = \frac{1}{S_1^3}(S_2^2 - S_1 S_3)$$
$$U_4 = \frac{1}{S_1^4}(-S_2^3 + 2S_1 S_2 S_3 - S_1^2 S_4)$$
$$U_5 = \frac{1}{S_1^5}(S_2^4 - 3S_1 S_2^2 S_3 + S_1^2 S_3^2 + 2S_1^2 S_2 S_4 - S_1^3 S_5)$$
$$J_1 = (r^4 - 6k_2^2 r^2 + k_2^4)G_1 + (2r^2 - 2k_2^2)G_2 + G_3$$
$$J_2 = (-4k_2 r^2 + 4k_2^3)G_1 - 4k_2 G_2 \tag{19}$$

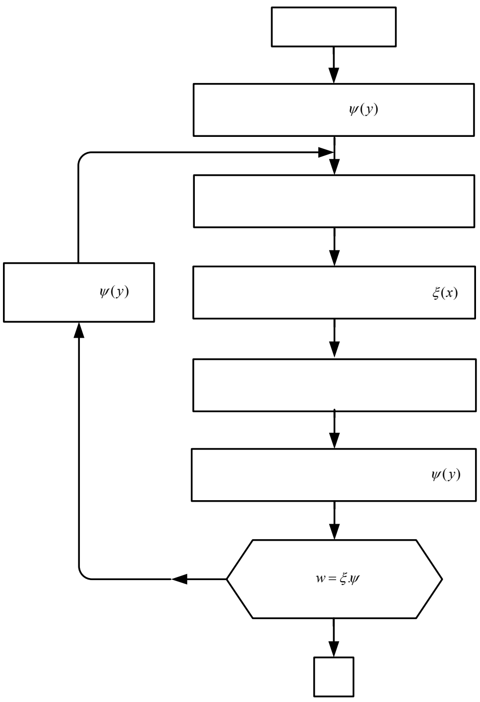

Fig. 2. Flowchart of procedure for the vibration analysis of orthotropic plates by EKM.

Furthermore, S's are defined as:

$$S_1 = F_1 k_1^4 - 2F_2 k_1^2 + F_3 \,, \quad S_2 = 4F_1 k_1^3 - 4F_2 k_1$$
$$S_3 = 6F_1 k_1^2 - 2F_2 \,, \quad S_4 = 4F_1 k_1$$
$$S_5 = F_1 \tag{20}$$

4. Results and discussions

Flowchart of the abovementioned procedure is shown in Fig. 2. A computer code in MATLAB is developed, which can be used for the vibration analysis of isotropic and orthotropic rectangular plates with different boundary conditions and general loading.

In the calculation, the first step is to assume an arbitrary function for $\psi(y)$. In contrast to other weighted residual methods, it does not need to satisfy boundary conditions since it will be satisfied in following iterations. Here the following form is used:

$$\psi = (y^2 - b^2)^2 \tag{21}$$

To demonstrate the applicability of the method, the different cases for isotropic plates ($\nu = 0.3$) and orthotropic plates with constants

$$D_2 / D_1 = 0.5 \,, \quad D_t / D_1 = 1/3 \,, \quad \nu_2 = 0.3$$

were solved for the following excitation loads.

Excitation loads, $q = q_0 Cos(\omega t)$

Tables 1 to 3 depict the non-dimensional displacement and bending moment amplitudes at the center of an isotropic simply-supported rectangular plate for several values of the aspect ratios. The results are compared with Romanelli and Laura [18].

Table 1. Non-dimensional deflection and bending moment, isotropic simply-supported plate ($b/a = 1$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4} \times 10^2$	$\frac{M}{q_0 l^2} \times 10^2$
0	[22]	0.4062	4.7886
	EKM	0.4058	4.7527
0.1	[22]	0.4104	4.8423
	EKM	0.4100	4.8063
0.3	[22]	0.4473	5.3142
	EKM	0.4469	5.2778
0.5	[22]	0.5448	6.5613
	EKM	0.5444	6.5242
0.7	[22]	0.8058	9.9041
	EKM	0.8054	9.8659
0.9	[22]	2.1796	27.524
	EKM	2.1792	27.4844
1.1	[22]	1.9916	26.004
	EKM	1.9921	26.046

Table 2. Non-dimensional deflection and bending moment, isotropic simply-supported plate ($b/a = 1.5$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4}\times 10^2$	$\frac{M_x}{q_0 l^2}\times 10^2$	$\frac{M_y}{q_0 l^2}\times 10^2$
0	[22] EKM	0.7724 0.7716	8.1160 8.0564	4.9843 4.9488
0.1	[22] EKM	0.7804 0.7797	8.2056 8.1459	5.0427 5.0072
0.3	[22] EKM	0.8511 0.8503	8.9938 8.9334	5.5567 5.5208
0.5	[22] EKM	1.0378 1.0370	11.077 11.0153	6.9177 6.8807
0.7	[22] EKM	1.5378 1.5370	16.663 16.599	10.575 10.536
0.9	[22] EKM	4.1713 4.1705	46.360 46.042	29.901 29.860
1.1	[22] EKM	3.8263 3.8271	43.360 43.432	28.891 28.934

Table 3. Non-dimensional deflection and bending moment, isotropic simply-supported plate (, $b/a = 2$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4}\times 10^2$	8×10^2	$\frac{M_y}{q_0 l^2}\times 10^2$
0	[22] EKM	1.0129 1.0119	10.168 10.099	4.6350 4.6041
0.1	[22] EKM	1.0235 1.0226	10.281 10.212	4.6916 4.6606
0.3	[22] EKM	1.1175 1.1165	11.274 11.203	5.1898 5.1581
0.5	[22] EKM	1.3659 1.3649	13.901 13.828	6.5136 6.4807
0.7	[22] EKM	2.0322 2.0312	20.956 20.879	10.091 10.056
0.9	[22] EKM	5.5466 5.5456	58.224 58.142	29.111 29.073
1.1	[22] EKM	5.1356 5.1367	55.140 55.229	28.957 28.998

Table 4. Non-dimensional deflection and bending moment, isotropic simply-supported plate.

$\frac{b}{a}$		$\frac{Dw}{q_0 l^4}\times 10^2$	$\frac{M_x}{q_0 l^2}\times 10^2$	$\frac{M_y}{q_0 l^2}\times 10^2$
1	FEM [23] Analytical [23] EKM	0.406 0.408 0.4058	4.790 4.944 4.7527	4.790 4.944 4.7527
1.5	FEM [23] Analytical [23] EKM	0.772 0.778 0.7716	8.118 8.310 8.0564	4.985 5.229 4.9488
2	FEM [23] Analytical [23] EKM	1.013 1.023 1.0129	10.117 10.385 10.099	4.635 4.897 4.6041

In Tables 4 to 6, the static solution ($\omega = 0$) for a similar case with three different boundary conditions and aspect ratios were given. The results are in very good agreement with the literature [19]. A comparison between the present study with literature [18] for non-dimensional displacement and bending moments at center of an orthotropic simply-supported plate is shown in Tables 7 to 9. As it can be seen both results are much close.

Table 5. Non-dimensional deflection and bending moment, isotropic SCSC plate.

$\frac{b}{a}$		$\frac{Dw}{q_0 l^4}\times 10^2$	$\frac{M_x}{q_0 l^2}\times 10^2$	$\frac{M_y}{q_0 l^2}\times 10^2$
1	FEM [23] Analytical [23] EKM	0.192 0.193 0.1913	2.440 2.571 2.3983	3.327 3.380 3.2722
1.5	FEM [23] Analytical [23] EKM	0.533 0.533 0.5318	5.850 5.949 5.7793	4.596 4.584 4.5455
2	FEM [23] Analytical [23] EKM	0.844 0.842 0.8434	8.690 8.694 8.605	4.737 4.602 4.6953

Table 6. Non-dimensional deflection and bending moment, isotropic clamped plate.

$\frac{b}{a}$		$\frac{Dw}{q_0 l^4}\times 10^2$	$\frac{M_x}{q_0 l^2}\times 10^2$	$\frac{M_y}{q_0 l^2}\times 10^2$
1	FEM [23] Analytical [23] EKM	0.126 0.126 0.1264	2.292 2.280 2.2630	2.292 2.280 2.263
1.5	FEM [23] Analytical [23] EKM	0.220 0.219 0.2192	3.681 3.659 3.6183	2.027 1.973 1.9928
2	FEM [23] Analytical [23] EKM	0.253 0.250 0.2525	4.120 4.045 4.032	1.581 1.431 1.5445

Table 7. Non-dimensional deflection and bending moment, orthotropic simply-supported plate ($b/a = 1$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4}\times 10^2$	$\frac{M_x}{q_0 l^2}\times 10^2$	$\frac{M_y}{q_0 l^2}\times 10^2$
0	[22] EKM	0.4714 0.4709	5.5558 5.5155	3.3462 3.3222
0.1	[22] EKM	0.4763 0.4758	5.6182 5.5780	3.3846 3.3605
0.3	[22] EKM	0.5193 0.5188	6.1676 6.1268	3.7220 3.6976
0.5	[22] EKM	0.6328 0.6323	7.6198 7.5878	4.6143 4.5892
0.7	[22] EKM	0.9368 0.9363	11.513 11.4696	7.0081 6.9820
0.9	[22] EKM	2.5373 2.5367	32.040 31.9933	19.637 19.6089
1.1	[22] EKM	2.3225 2.3231	32.040 30.3751	19.637 18.7755

Table 8. Non-dimensional deflection and bending moment, orthotropic simply-supported plate ($b/a = 1.5$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4} \times 10^2$	$\frac{M_x}{q_0 l^2} \times 10^2$	$\frac{M_y}{q_0 l^2} \times 10^2$
0	[22] EKM	0.8151 0.8144	8.5162 8.4570	3.6729 3.6477
0.1	[22] EKM	0.8237 0.8230	8.6113 8.5520	3.7163 3.6911
0.3	[22] EKM	0.8988 0.8981	9.4482 9.3879	4.0983 4.0726
0.5	[22] EKM	1.0974 1.0967	11.662 11.600	5.1108 5.0842
0.7	[22] EKM	1.6297 1.6290	17.603 17.538	7.8370 7.8088
0.9	[22] EKM	4.4352 4.4344	48.966 48.900	22.271 22.242
1.1	[22] EKM	4.0878 4.0886	46.392 46.466	21.691 21.723

Table 9. Non-dimensional deflection and bending moment, orthotropic simply-supported plate ($b/a = 2$).

$\frac{\Omega}{\Omega_1}$		$\frac{Dw}{q_0 l^4} \times 10^2$	$\frac{M_x}{q_0 l^2} \times 10^2$	$\frac{M_y}{q_0 l^2} \times 10^2$
0	[22] EKM	1.0318 1.0309	10.309 10.243	3.7426 3.7187
0.1	[22] EKM	1.0428 1.0418	10.424 10.358	3.7874 3.7632
0.3	[22] EKM	1.1393 1.1379	11.441 11.370	4.1821 4.1558
0.5	[22] EKM	1.3946 1.3920	14.137 14.049	5.2319 5.1993
0.7	[22] EKM	2.0804 2.0724	21.391 21.241	9.0738 8.0172
0.9	[22] EKM	5.7036 5.6207	59.798 58.848	23.219 22.846
1.1	[22] EKM	5.3204 5.4248	57.213 58.401	23.100 23.566

Excitation loads

$$q = q_0(x^4 + x^3 + x^2 + x + 1)Sin(5y)Cos(\omega t)$$

In this example, an orthotropic clamped plate with aspect ratio of 1.5 was considered. Non-dimensional deflection ($w^* = D_1 w / q_0 l^4$) and bending moments ($M^* = M / q_0 l^2$) of the plate along the centerlines (*l/2, y*) and (*x*, *b/2*) are presented in Figs. 3 to 8.

In the other case, the same type of plate as the previous example is applied. For the case of SSCC boundary conditions, non-dimensional deflection (w^*) and bending moments (M^*) of the plate along the centerlines (*l/2, y*) and (*x, b/2*) are depicted as Figs. 9 to 14.

In the tables and figures, *Ω* is the frequency parameter and is defined as the following:

For isotropic: $\Omega = \omega a^2 \sqrt{\rho h / D}$

For orthotropic: $\Omega = \omega a^2 \sqrt{\rho h / D_1}$ (22)

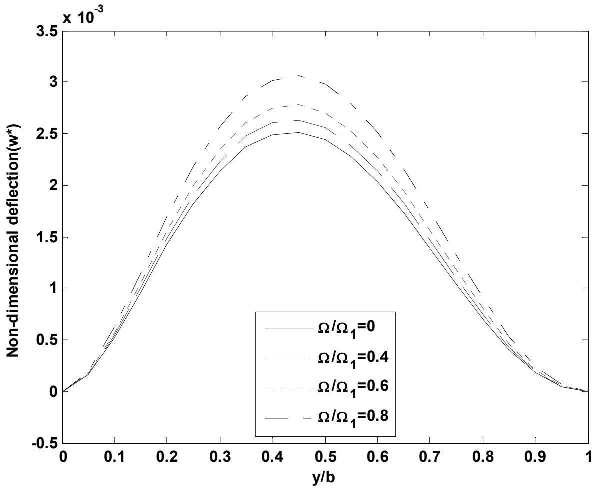

Fig. 3. Non-dimensional deflection (w^*) of a clamped plate along centerline (*l/2, y*).

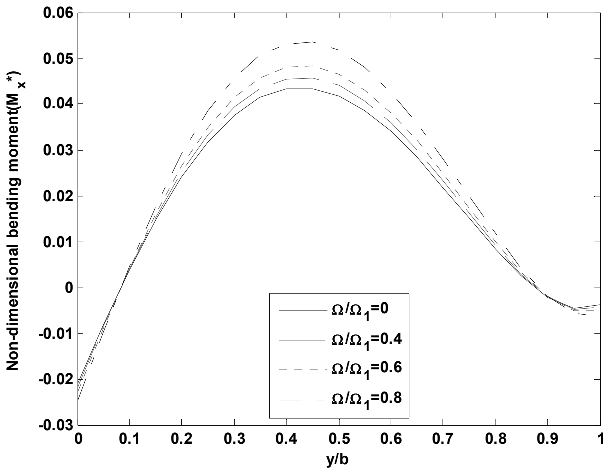

Fig. 4. Non-dimensional bending moment ($M_x{}^*$) of a clamped plate along centerline (*l/2, y*).

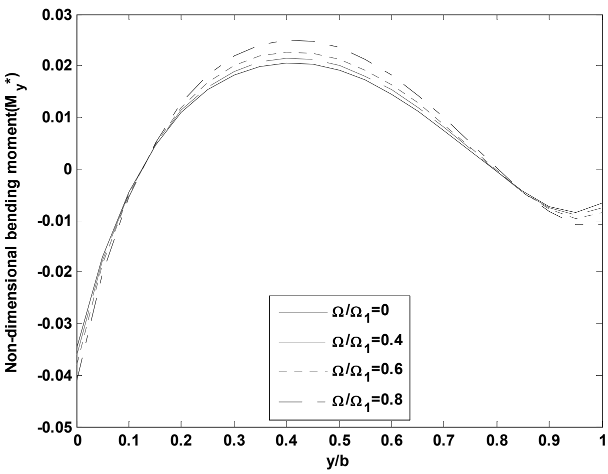

Fig. 5. Non-dimensional bending moment ($M_y{}^*$) of a clamped plate along centerline (*l/2, y*).

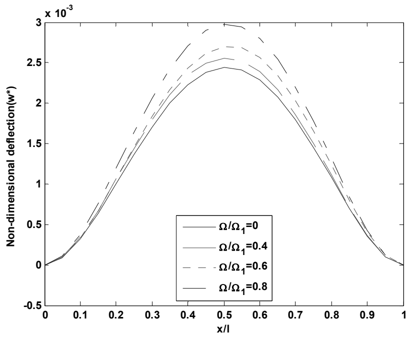

Fig. 6. Non-dimensional deflection (w^*) of a clamped plate along centerline (*x, b/2*).

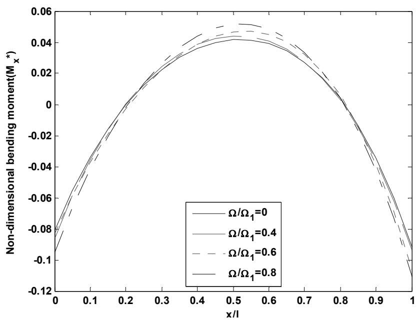

Fig. 7. Non-dimensional bending moment (M_x*) of a clamped plate along centerline (*x, b/2*).

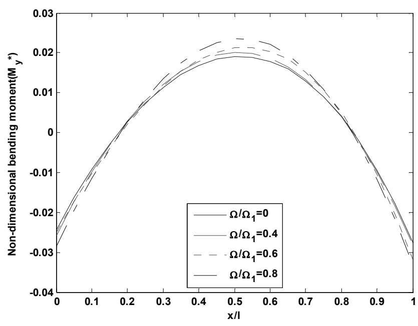

Fig. 8. Non-dimensional bending moment (M_y*) of a clamped plate along centerline(*x, b/2*).

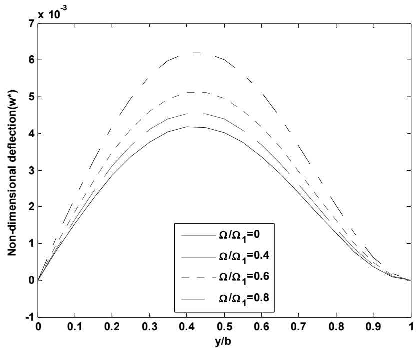

Fig. 9. Non-dimensional deflection (w^*) of SSCC plate along centerline (*l/2, y*).

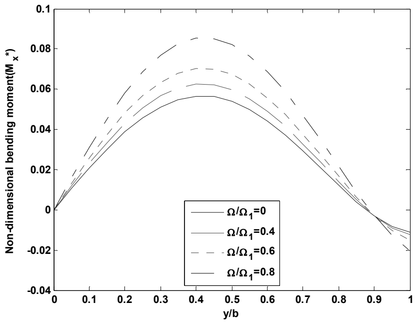

Fig. 10. Non-dimensional bending moment (M_x *) of SSCC plate along centerline (*l/2, y*).

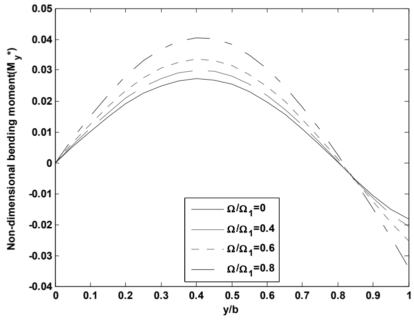

Fig. 11. Non-dimensional bending moment (M_y *) of SSCC plate along centerline (*l/2, y*).

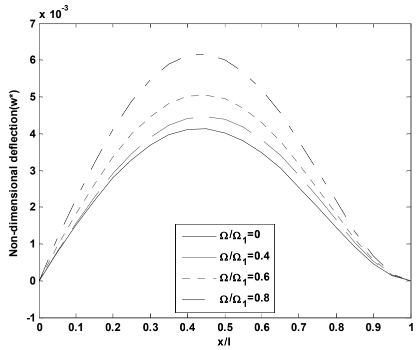

Fig. 12. Non-dimensional deflection (w^*) of SSCC plate along centerline (*x, b/2*).

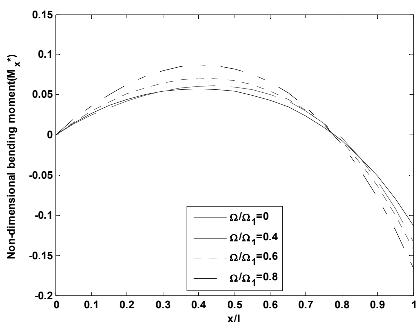

Fig. 13. Non-dimensional bending moment (M_x *) of SSCC plate along centerline (*x, b/2*).

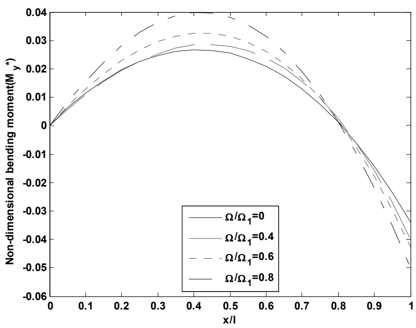

Fig. 14. Non-dimensional bending moment (M_y *) of SSCC plate along centerline (*x, b/2*).

5. Concluding remarks

The extended Kantorovich method is successfully applied to obtain highly accurate approximations of the closed-form solution for the forced vibration analysis of orthotropic rectangular plates.

The boundary conditions of the plate can be any combination of simply-supported and clamped edge. In this analysis, the plate is subjected to a general non-uniform distributed dynamic load. Application of EKM to the system of partial differential equations reduces the governing equations of the problem to a double set of ordinary differential equations. These sets of equations were then solved in an iterative manner until convergence was achieved. In any iteration, exact closed form solutions were obtained for ODE systems. The most significant feature of the present method is that the iteration procedure is insensitive to the initial guess for the solution and satisfies all boundary conditions in subsequent iterations. Dynamic assessment of orthotropic plates employing EKM requires much less effort than the other approximate techniques, such as the finite element method, because the major portion of the computation is done analytically. Furthermore, convergence was fast, as judged by the number of iterations, which in no cases exceeded three. In this study, a computer code using MATLAB was prepared. Dynamic displacement and the bending moment amplitudes were determined in no time, and the results obtained for the different cases was in very good agreement with the available published studies.

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